EFFECTS OF SOIL PLUG ON BEHAVIOUR OF DRIVEN PIPE PILES

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ABSTRACT

Stress wave theory is applied to open-ended pipe piles to clarify the effects of soil plug on the behaviour of piles during driving and static loading. Measured field data and various numerical models are reviewed; methods are presented to calculate wave propagation in both the pile and the soil plug; modelling is presented which takes into account the interaction between the soil plug and the pile; also presented is simplified method to estimate the load-settlement relation of the pipe pile in static loading. By correlating observed and calculated values in two analytical cases, the authors demonstrate that incorporation of the soil plug (modelled as a series of masses and springs) is required to correctly predict pile behaviour during driving and static loading.

Key words: bearing capacity, case history, friction, load test, pile driving, pipe pile, soil plug, wave propagation (JGC: K7/E4/E8)

INTRODUCTION

Application of stress wave theory to pile driving has received an increasing attention in many aspects of pile design and construction, including the estimation of current total soil resistance, hammer efficiency, pile integrity, and the estimation of pile drivability and static bearing capacity. The practical aspect of this trend may be associated with the increasing use of deep penetration piles for offshore structures (McClelland, 1974).

In this paper, stress wave theory is applied to the behaviour of open-ended pipe piles during driving. Related studies including measured field data and various numerical models are reviewed. A method for analyzing pile driving of a pipe pile is presented in this paper based on the work of Randolph and Simons (1986) which takes into account the interaction between the soil plug and the pile. Also presented is a simplified method to estimate the load-settlement relation of the pipe pile in static loading using the same modelling of a pipe pile that is used for pile driving. Pile driving and static loading tests of an instrumented, offshore steel pipe pile driven with a diesel hammer are analyzed based on the methods

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developed here. Pile driving analysis is combined with the two-point strain measurement method permitting us to bypass modelling of the hammer action including gas forces and cushions. It is shown that the analyzed behaviour of the pile during driving and in static loading conforms well with the measured pile performance.

**REVIEW OF RELATED STUDIES**

*Dynamic Measurements and Numerical Models*

Dynamic wave measurements at the pile head during driving allows us to estimate at any given moment the current total resistance of the soil to pile penetration, including both shaft and base resistance. Such measurements also permit us to calculate the energy transmitted to the pile from the hammer, without modelling the soil response and the hammer. The Case method (Goble et al., 1975; Rausche et al., 1985) and the method of two-point strain measurements (Lundberg and Henchoz, 1977; Lundberg, 1984; Matsumoto et al., 1988) represent this type of application of stress wave theory. Measured dynamic data also permit the estimation of pile integrity.

On the other hand, numerical modelling of hammer, pile and soil response is required to assess the drivability of a pile and to estimate the distribution and magnitude of the static soil resistance along the pile shaft and at the base of the pile by correlation of stress wave data. If one-dimensional modelling of hammer, pile and soil is used in the analysis of pile driving, the soil response is usually represented by a spring and slider in conjunction with a dashpot to account for differences in soil response between dynamic and static loading (Smith, 1960). Conceptually, the spring and slider represent the static soil response, whereas the dashpot is understood as the dynamic component of soil resistance due to viscous (or material) damping.

In Smith’s soil model, coefficients of viscous damping for various soils have been recommended empirically so that the static bearing capacity derived from stress wave data correlation is identical to the bearing capacity determined by static loading tests carried out after driving (Goble et al., 1984). On the contrary, Novak et al. (1977) propose an approach to determine rationally the damping constant of the shaft resistance, where the dashpot represents radiation damping. The approach for modelling radiation damping accurately has been suggested by Meynard and Corte (1984), Randolph and Simons (1986), Randolph (1987), and Chow et al., (1988). A revised soil model has been proposed by Randolph and Simons (1986), in which the contribution of radiation damping and the viscous damping to the dynamic soil resistance are separately considered. Nguyen et al. (1988) proposed a shaft soil model which takes into account the hysteretic damping and the reduction of the shear modulus associated with the increase in the shear strain around the pile. Mitwally and Novak (1988) also proposed a shaft model which included factors for the weakened zone around the pile. These soil models estimate the values of radiation damping smaller than those proposed by Novak et al. (1977). Middendorp and Brederode (1984) proposed a skin friction model in which an added soil mass is combined with the original Smith model to account for acceleration-dependent frictional forces. Lysmer and Richart (1966) proposed a spring–dashpot analogue in which the dashpot represented radiation damping and in which the vertical vibration of a rigid circular footing on the surface of an elastic halfspace may represent the dynamic soil response at the base of a solid pile.

Reliability of numerical solutions depends greatly on the choice of an appropriate soil model and soil parameters, as well as how the hammer–pile–soil system is modelled. This is particularly true for open-ended pipe piles where shaft resistance at the pile–soil interface between the soil plug and the internal lateral surface of the pile causes stress waves up and down in the soil plug as well as in the pile. In the usual one-dimensional analysis of pile driving, shear stresses along the external and the internal pile surfaces are expressed as frictional forces acting on pile nodes in such way that mobilization of the internal shear stress
is identical to that of the external shear stress. However, in actuality, the mobilization of the internal shear stress can be expected to differ from that of the external shear stress due to the infinite radial extent of the surrounding ground versus the one-dimensional configuration of the soil column inside the pile. For a friction pipe pile, the vertical shear deformation will be predominant in the surrounding ground, associated with shear waves propagating in a radial direction. Shear stresses along the interface between the soil plug and the internal lateral surface of the pile also cause vertical shear waves propagating in a radial direction towards the central axis of the soil plug inside the pile. Vertical shear waves in the soil plug will cause stress waves up and down the soil plug because of the cylindrical configuration of the soil plug. Wave propagation through the soil plug can be expressed adequately as longitudinal wave propagation.

**Observed Behaviour of Soil Plug During Driving**

Nishida et al. (1985a) reported that the top surface of the soil inside the pile rose up to 0.72 m below the original ground surface after driving when a steel pipe pile having an outer diameter of 406 mm and a wall thickness of 11.9 mm was driven with a diesel hammer into a diatomaceous mudstone to a depth of 10 m. The same authors also reported that the soil plug ascended 1.45 m above the original ground surface when a steel pipe pile with an outer diameter of 1200 mm and a wall thickness of 14 mm was driven into the same ground to a depth of 22 m.

Morita et al. (1989) measured the soil plug heights after driving of 111 steel pipe piles into a mixed substrata composed of alternating layers of reclamation soil, clay and gravel. The piles had a diameter of 1016 mm and were driven with a hydraulic hammer. The measured levels of the top surfaces of the soil plugs ranged from 0.5 m to 6 m below the original ground surface with an average depth of 3 m, even though all piles were driven to a depth of 31 m.

Niyama et al. (1989) recorded in detail the relationship between the height of the soil plug and the embedded pile length during driving of two open-ended concrete piles driven with an air steam hammer into stratified sands, interbedded erratically with layers of silty clay. Both concrete piles were 41 m long, with a 90 cm outer diameter and a 11.5 cm wall thickness. One of them was coated with asphalt in order to minimize the negative friction effect, whereas the other was not treated. The height of the soil plug during driving were almost equal to the penetration depths for both piles, implying that the top surface of the soil plug was kept at the original ground surface throughout driving.

**Observed Behaviour of Soil Plug in Static Loading**

Behaviour of the soil plug in static loading tests is different from that during driving. Kishida (1967) conducted model pile loading tests on 62 open-ended steel pipe piles in sand. The test piles had a length of 50 cm and outer diameters ranging from 10 mm to 100 mm, with a diameter to wall thickness ratio of 55:1. The model ground was dry Toyoura sand having a unit weight of 1.37 g/cm³ (13.5 kN/m³), a void ratio of 0.912, and an internal friction angle of 26 degrees. The friction angle between the sand and the steel pile was 16 degrees. The piles were pushed into the ground with a loading rate of 2 mm/s, and the penetration depth, D, and the height of the soil, H, inside the piles were successively recorded.

The experiments showed that the rate of increase in the height of the soil plug suddenly decreased when the ratio of the height of the soil plug to the penetration depth, H/D, reached 2 to 5, irrespectively of the pile diameters. Nishida et al. (1985b) also observed similar behaviour of the soil plug inside a model steel pipe pipe having an outer diameter of 22 mm which was jacked into a diatomaceous mudstone ground. These experimental data show that pipe piles are “perfectly” plugged in static loading tests, although the plugging of pipe piles in static loading tests should be understood in terms of the interactions among the pile, the soil inside the pile, the bearing
capacity of the ground at the base of the pile, as well as the external soils.

Inclusion of Soil Plug in Pipe Pile Modelling

The field evidence and experimental data mentioned above clearly show a difference in soil plug behaviour during driving and in static loading. This may be attributable to the difference in load transfer between dynamic and static conditions. In cases when the assessment of drivability of a pipe pile or estimation of the pile behaviour in static loading is made from the analysis of pile driving, the modelling of pile driving which accurately represents the load transfer during driving and static loading, as well as the choice of an appropriate soil model and soil parameters, is required. Heerema and de Jong (1980) have proposed an approach to model such behaviour of the soil plug and the interaction between the soil plug and the pile during driving. Randolph and Simons (1986) have revised the approach proposed by Heerema and de Jong.

THEORETICAL BASES

Pipe Pile Model

The pile–soil model for the steel pipe pile used in the analysis is shown in Fig. 1. The soil plug is modelled as a series of masses and springs, namely the “pile within a pile” proposed by Heerema and de Jong (1979), with frictional forces between the soil nodes and pile nodes, so that the interaction between the soil plug and the pile can be introduced in the analysis of pile driving. In this pile driving analysis, stress waves propagating up and down the soil plug are calculated based on Smith’s method (1955), whereas the characteristic solutions of the wave equation are adopted to calculate wave propagation in the pile. This type of analysis of pile driving was used by Randolph and Simons (1986) and Rondolphi (1987). Takei, Matsumoto, Nishida and Motoyama (1989) have developed a computer program based on the above mentioned pile–soil model.

Compressive stress and compressive strain are taken as positive throughout the present article. The positive direction of the coordinate, $x$, is taken downward along the pile axis.

Soil Model

The soil response used in the present analysis for the internal and external shear stresses at pile–soil interfaces is that proposed by Randolph and Simons (1986) and shown in Fig. 2(a). The slider represents the static maximum shear stress, $\tau_{\text{max}}$. The dashpot in the upper portion of the model represents the viscous damping due to the relative velocity between the pile and the soil adjacent to the pile, whereas the dashpot in the lower portion represents radiation damping of the soil. If the viscous damping is negligible, the response of the external shear stress, $\tau_{\text{out}}$, is expressed by

\[ \tau_{\text{out}} = k_s u_s + c_s \dot{u}_s \leq \tau_{\text{max}} \quad (1) \]

where $u_s$ is the displacement and $\dot{u}_s$ is the velocity of the soil adjacent to the pile, $k_s$ is the spring stiffness and $c_s$ is the radiation damping constant for the external shear stress. Relative displacements between the pile, $u$, and the soil, $u_s$, adjacent to the pile do not occur ($u = u_s$) until the maximum shear stress, $\tau_{\text{max}}$, is reached.

The external frictional stresses which act along a pile segment having a length of $JL$
are converted to a frictional force, \( r_m \), acting on the \( m \)-th node of the pile:

\[
\begin{align}
\tau_{m} &= k_s \times (U^1 - U^2) + c_s \times (\dot{U}^1 - \dot{U}^2) \\ 
\leq 2\pi r_d dL \sigma_{\text{max}} \\
\end{align}
\]

\[
\begin{align}
R_m &= D_m \times (U_m^1 - U_m^2) + J_m \times (\dot{U}_m^1 - \dot{U}_m^2) \\
\leq 2\pi r_i dL \sigma_{\text{max}} \\
\end{align}
\]

where \( r_i \) is the inner radius of the pile, and \( U^1 \) is the displacement of the centre of the soil plug, \( U^2 \) is the displacement of the soil plug adjacent to the pile, and \( c_s \) is the radiation damping constant for the internal shear stress.

The conventional soil model suggested by Smith (1960) (Fig. 2(b)) is adopted to represent the response, \( \sigma_{bs} \), at the base of the pile:

\[
\sigma_{bs} = k_b \times u_n + c_b \times \dot{u}_n
\]

where \( u_n \) is the displacement of the pile base \( (m=n) \), \( k_b \) is the spring stiffness, and \( c_b \) is the damping constant for the pile base. Therefore, the base force, \( r_n \), is given by

\[
\begin{align}
r_n &= d_n \times u_n + f_n \times \dot{u}_n \\
d_n &= A \times k_b \\
f_n &= A \times c_b
\end{align}
\]

where \( A \) is the cross-sectional area of the pile.

The soil reaction, \( \sigma_{bs} \), at the base of soil plug is similarly expressed by

\[
\sigma_{bs} = k_b \times U_n^2 + c_b \times \dot{U}_n^2
\]

where \( U_n^2 \) is the displacement of the soil plug at the level of the pile base, \( k_b \) is the spring stiffness, and \( c_b \) is the damping constant of the base of the soil plug. Then, the point-bearing force, \( R_n \), of the soil plug is given by

\[
\begin{align}
R_n &= D_n \times U_n^2 + J_n \times \dot{U}_n^2 \\
D_n &= A_x \times K_b \\
J_n &= A_x \times C_b
\end{align}
\]

where \( A_x \) is the cross-sectional area of the soil plug.

**Calculation of Wave Propagation in The Pile**

The characteristic solutions for the wave equation are used to calculate wave propagation in the pile. Here, let \( m, f_m, g_m \) and \( v_m \) be the displacement, downward travelling stress
Fig. 3. Notations used in calculation of wave propagation in pile

wave, upward travelling stress wave, and particle velocity, respectively, at the \( m \)-th node of the pile. And let \( \sigma_m \) be the stress of the \( m \)-th element of the pile. If these quantities are known at time \( t = t - \Delta t \), then the quantities at time \( t = t \) are calculated by using the following equations (Fig. 3): 

\[
\begin{align*}
    u_m(t) &= u_m(t - \Delta t) + v_m(t - \Delta t) \cdot \Delta t \\
    f_m(t) &= f_{m-1}(t - \Delta t) - F_m(t) / 2A \\
    g_m(t) &= g_{m+1}(t - \Delta t) + F_m(t) / 2A \\
    \sigma_m(t) &= f_{m-1}(t - \Delta t) + g_{m+1}(t - \Delta t) \\
    v_m(t) &= (c/E) \cdot [f_{m-1}(t - \Delta t) - g_{m+1}(t - \Delta t) - F_m(t) / 2A]
\end{align*}
\]

Here, \( F_m \) is the frictional force acting on the \( m \)-th node of the pile and is equal to the sum of the external frictional force, \( r_m \), and the internal frictional force, \( R_m \). The forces, \( r_m \), \( R_m \) and \( F_m \), acting upwards to the pile are taken as positive, \( c \) is the bar wave velocity, and \( E \) is Young's modulus of the pile. The time interval, \( \Delta t \), is given by 

\[
\Delta t = \Delta L / c
\]

Note that Eq. (19) is replaced by the following equation for the pile base (\( m = n \)): 

\[
g_n(t) = -[f_{n-1}(t - \Delta t) - r_n(t) / A]
\]

Calculation of Wave Propagation in The Soil Plug

Fig. 4. Notations used in calculation of wave propagation in soil plug

Wave propagation in the soil plug is calculated based on Smith's method (1955). The mass, \( M_m \), and the spring constant, \( B_m \), which represent the column of the soil plug, are given as follows: 

\[
M_m = \rho_s A_s \Delta L \\
B_m = \Delta L E_{st} / \Delta L
\]

where \( \rho_s \) is the mass density of the soil plug. The constrained Young's modulus, \( E_{st} \), is used to represent the spring constant of the soil columns, since the soil plug may undergo only vertical deformation without the lateral deformation due to a relatively large rigidity of the pile.

Wave propagation through the soil plug is calculated using the following equations (Fig. 4): 

\[
\begin{align*}
    U_m^2(t) &= U_m^2(t - \Delta t) + V_m^2(t - \Delta t) \cdot \Delta t \\
    C_m(t) &= U_{m-1}^2(t) - U_m^2(t) \\
    N_m(t) &= B_m \cdot C_m(t)
\end{align*}
\]
\[ Z_m(t) = N_m(t) - N_{m+1}(t) + R_m(t) \]  \hspace{1cm} (29) \\
\[ V_m(t) = V_m(t - \Delta t) + Z_m(t) \times (\Delta t \cdot g/M_m) \]  \hspace{1cm} (30)

Here, \( U_m^i, R_m, Z_m \) and \( V_m^i \) are the displacement, internal frictional force, accelerating force and particle velocity, respectively, at the \( m \)-th node of the soil plug; \( C_m \) and \( N_m \) are the compression and the axial force of the \( m \)-th element of the soil plug, and \( g \) is the acceleration of gravity. Note that Eq. (29) is replaced by the following equations for the top surface \((m=s)\) and the bottom \((m=n)\) of the soil plug:

\[ Z_s(t) = -N_{s+1}(t) + R_s(t) \]  \hspace{1cm} (31) \\
\[ Z_n(t) = N_n(t) - R_n(t) \]  \hspace{1cm} (32)

The frictional forces, \( r_m, R_m \) and \( F_m \) are calculated at each timestep. When yielding occurs at the external pile surface, the change in displacement, \( du \), of the external soil occurring as the stress, \( \tau_{\text{max}} \), acts over the time interval, \( \Delta t \), is calculated as follows (Simons and Randolph, 1985):

\[ du = (Q - u^i) \left[ 1 - \exp \left( -\frac{k_s \Delta t}{c_s} \right) \right] \]  \hspace{1cm} (33)

where \( Q = \tau_{\text{max}}/k_s \), and \( u^i \) is the soil displacement at the start of the timestep. The rejoining of the pile and the external soil is achieved when the sign of \((\dot{u} - \dot{u}^i)\) at the present timestep changes from that at the previous timestep, or when the absolute value of \( \tau \)

\[ \tau = k_s \times (U^i - U) + c_s \times (\dot{U} - \dot{U}^i) \]  \hspace{1cm} (36)

falls below the maximum shear stress, \( \tau_{\text{max}} \).

**Simplified Method to Calculate The Load–Settlement Curve**

Behaviour of the soil plug inside a pipe pile in a static loading test is complicated, since the behaviour of the soil plug is affected by inner diameter of the pile, expansive deformation of the pile, dilation of the soil inside the pile, arching effects developed in the soil plug, and so forth resulting in non-uniform radial distribution of the vertical stress in the soil plug (for example, Nagai, 1985). However, the pile–soil model shown in Fig. 1 can be used to calculate the approximate load–settlement curve in a static loading test. A matrix method is used to calculate the displacement, axial force and shear stresses at the pile–soil interfaces at each point of the pile. For further explanation of this method, refer to the APPENDIX.

**ANALYSIS OF OFFSHORE TEST PILING**

**Outline of The Offshore Test Piling Discussed**

Outline of the offshore pile driving and loading test were summarised by Motoyama et al. (1988). Shibata et al. (1989) analyzed in detail the wave propagation in the pile by using the method of two-point strain measurements. It is appropriate to review these works for the discussion which follows.

The offshore pile driving test was performed in 1986 in association with the design of foundation piles for the access bridge that spans the Kansai International Airport island in Osaka Bay, 5 km offshore, and the mainland (Fig. 5). The airport island and access bridge are currently under construction with the scheduled start of operation in 1993.

The stratification of the submarine deposit along the axis of the access bridge is indicated in Fig. 6. The soil conditions become less conducive to pile driving as the airport island is approached, owing principally to the increased
**Table 1. Test pile dimensions**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>$L$</td>
<td>58</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>$D_o$</td>
<td>1500</td>
</tr>
<tr>
<td>Inner diameter</td>
<td>$D_i$</td>
<td>1456</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>$T$</td>
<td>22</td>
</tr>
<tr>
<td>Sectional area†</td>
<td>$A$</td>
<td>1171</td>
</tr>
<tr>
<td>Mass density</td>
<td>$\rho$</td>
<td>7.611</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E$</td>
<td>$2.1 \times 10^4$</td>
</tr>
<tr>
<td>Wave velocity</td>
<td>$c$</td>
<td>5200</td>
</tr>
</tbody>
</table>

†: including the sectional area of protector

Of the three test pileings carried out, we will discuss only the test piling designated as $T_1$. Open-ended steel pipe piles were used, and the dimensions of the test pile are listed in Table 1. The test pile was instrumented with strain gauges (WFLA-6-1L) at a total of 12 sections, as shown in Fig. 7. Note that at each section of strain measurement, four strain gauges were mounted 90 degrees apart on the inner surface of the pile. Note also that protection of the strain gauges with steel channels increased the cross-sectional area, $A$, of the test pile to 1171 cm$^2$.

The hammer used was a diesel hammer (MH-72B) that had a ram weighing 7.2 tf (70.6 kN). A cushion comprised of synthetic rubbers spaced with thin steel plates, was set between the anvil and the pile cap. The test pile was driven to the relatively thin stratum of dense sand designated as $S_1$ (Fig. 7). The bounce and set records at the last ten blows showed that the permanent set per blow, $S$, was equal to 2 mm, whereas the rebound per blow, $K$, amounted to 7 mm. The corresponding stroke, $h$, of the hammer was equal to 2.39 m on the average. Note that the level of the top surface of the soil inside the pile

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**Fig. 5. Airport island and access bridge in Osaka Bay, Japan** (Shibata et al., 1989)

**Fig. 6. Soil stratification along the access bridge** (Shibata et al., 1989)
rose up 1.1 m above the original surface of the seabed after driving (see Fig. 7), despite of a relatively deep embedment depth of 37.1 m.

The stress waveforms recorded during the last driving are shown in Fig. 8; they were measured with a sampling time of 0.1 ms and stored as digital data. The behaviour of the test pile during driving was monitored and analyzed by using the method of two-point strain measurements (Shibata et al., 1989). The feature of the method of two-point strain measurements is that the method enables determination of strain, particle velocity and energy transmission at an arbitrary section of a pile based on strain measurements at two different sections of a pile (Lundberg and Henchoz, 1977; Matsumoto et al., 1988). The stress waveforms selected were those measured at gauge points No. 20 and No. 17 above the ground surface. The monitored movement of the pile head by the last blow is shown in Fig. 9. The monitored energy transmitted to the pile for the last blow is shown in Fig. 10. It was observed that the energy actually transmitted to the pile was 695 tf cm (68.1 kN m) which was only 45% of the potential energy of the hammer.

The static loading test, performed 35 days after the final driving, showed that the test pile had an ultimate, axial bearing capacity

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![Fig. 7. Test pile in final seating, together with ground conditions (Motoyama et al., 1988)](image)

![Fig. 8. Recorded stress waveforms at the last driving (1 kgf/cm² = 98 kPa)](image)

![Fig. 9. Movement of the pile head monitored by two-point strain measurements (Shibata et al., 1989)](image)
of 1300 tf (12.7 MN), as illustrated in Fig. 11. The measured axial force distributions in the static loading test are shown in Fig. 12. It can be seen that a larger portion of the bearing capacity was due to the shaft resistance and that the base resistance was only 300 tf (2.9 MN).

The shear stress, \( \tau \), of each stratum mobilized with the displacement, \( u \), of the pile at the level of the midpoint of the corresponding stratum is shown in Fig. 13. Note here that the shear stress, \( \tau \), was calculated assuming that such stress acted only along the outer lateral surface of the pile, although shear stress along the inner surface of the pile actually

![Fig. 10. Energy transmission monitored by two-point strain measurements (1 tf cm = 98 Nm) (Shibata et al., 1989)](image)

![Fig. 11. Pile head force-displacement curves in static loading test (1 tf = 9.8 kN) (Shibata et al., 1989)](image)

![Fig. 12. Axial force distribution in static loading test (1 tf = 9.8 kN) (Kansai Int. Airport Co., 1987)](image)

![Fig. 13. Mobilization of shear stress estimated from static loading test (1 kgf/cm² = 98 kPa) (Kansai Int. Airport Co., 1987)](image)
seemed to be developed as well, judging from the existence of the soil inside the pile after driving (Fig. 7). It was observed that the shear stress, \( \tau \), of all layers reached peak values when the maximum load of 1300 tf (12.7 MN) was applied to the pile head, although the shear stress of the dense sand (S4) exhibited a post-peak, strain-softening behaviour.

**Determination of Soil Parameters**

In this analysis, measured values were as follows: mass density \( \rho_s \), shear wave velocity \( V_s \), and primary wave velocity \( V_p \), of the soils at the test site. The shear modulus \( G_s \), the constrained Young’s modulus \( E_{cs} \), and the poisson’s ratio \( \nu_s \), were estimated using the following relations from the theory of elasticity and summarized in Table 2:

\[
G_s = \rho_s V_s^2 \\
E_{cs} = \rho_s V_p^2 \\
\nu_s = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)} = \frac{E_{cs} - 2G_s}{2(E_{cs} - G_s)}
\]

Note that some values of the soil parameters were assumed, due to the lack of measured data (see Table 2).

There are some analytical approaches available to estimate soil parameters such as \( k_s \), \( K_s \), \( c_s \) and \( C_s \), from measured, conventional soil parameters. Novak et al. (1978) presented an analytical solution for the dynamic soil reaction at the external lateral surface of the infinitely long rigid pile embedded in an elastic soil undergoing harmonic vibration in the direction of the pile axis. The spring stiffness, \( k_s \), and the radiation damping constant, \( c_s \), are approximated, independently of the vibration frequency, as follows:

\[
k_s = 2.75 \frac{G_s}{\pi r_0} \tag{40}
\]

\[
c_s = \frac{G_s}{V_s} \tag{41}
\]

The coefficients, \( K_s \) and \( C_s \), of the spring-dashpot analogue (Fig. 2 (b)) for the soil reaction at the base of the soil plug are approximated as follows (Lysmer and Richart, 1966):

\[
K_s = \frac{4G_s}{\pi r_0 (1 - \nu_s)} \tag{42}
\]

\[
C_s = \frac{3.4}{\pi (1 - \nu_s)} \sqrt{\rho_s G_s} \tag{43}
\]

The soil parameters representing the soil reaction used in the analyses of pile driving and static loading were determined from the results of the static loading test and the analytical approaches mentioned above.

Two separate analyses of pile driving and static load testing were carried out in this study: Case 1 and Case 2. The internal shear stress was assumed to be negligible in Case 1, whereas the internal shear stress and wave propagation in the soil plug were incorporated in the analysis of pile driving in Case 2. In Case 1, the distribution of the maximum shear stress, \( \tau_{\text{max}} \), along the external lateral surface was taken from the measured maximum shear stress (Fig. 13). It should be noted again that the measured maximum shear stress, \( \tau_{\text{max}} \), was evaluated assuming that shear stresses acted along only the external lateral surface of the pile. The maximum base resistance was also taken from the axial stress transmitted to the pile base at the final pile head force of 1300 tf (12.7 MN) in the static loading test. The spring stiffness, \( k_s \), for the external shear stress was estimated using Eq. (40), whereas the damping constant, \( c_s \), was taken as zero. The spring stiffness, \( k_b \), and the damping constant, \( c_b \), at the pile base were determined from Eq. (42) and Eq. (43), respectively, although the pile had an annular configuration which did not match the situation considered by Lysmer and Richart (1966). Soil parameters used in Case 1 are summarized in Table 3.

In Case 2, it was assumed that the distribution of the maximum shear stresses, \( \tau_{\text{max}} \), along

---

**Table 2. Measured soil parameters (1 kgf/cm² = 98 kPa)**

(Kansai Int. Airport Co., 1985)

<table>
<thead>
<tr>
<th>Layer</th>
<th>( \rho_s ) g/cm³</th>
<th>( V_s ) m/s</th>
<th>( V_p ) m/s</th>
<th>( G_s ) kgf/cm²</th>
<th>( E_{cs} ) kgf/cm²</th>
<th>( c_s ) (meas.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ac</td>
<td>1.46</td>
<td>80⁺</td>
<td>—</td>
<td>95</td>
<td>4863</td>
<td>0.49⁺</td>
</tr>
<tr>
<td>Tq</td>
<td>1.8</td>
<td>188</td>
<td>1680</td>
<td>684</td>
<td>51340</td>
<td>0.493</td>
</tr>
<tr>
<td>S1</td>
<td>1.84</td>
<td>182</td>
<td>1500</td>
<td>622</td>
<td>45345</td>
<td>0.493</td>
</tr>
<tr>
<td>C1</td>
<td>1.59</td>
<td>164</td>
<td>1490</td>
<td>436</td>
<td>36020</td>
<td>0.494</td>
</tr>
<tr>
<td>C2</td>
<td>1.81</td>
<td>239</td>
<td>1490</td>
<td>1055</td>
<td>41004</td>
<td>0.488</td>
</tr>
<tr>
<td>S4</td>
<td>1.8</td>
<td>291</td>
<td>1710</td>
<td>1555</td>
<td>53708</td>
<td>0.486</td>
</tr>
</tbody>
</table>

*Assumed
Table 3. Soil parameters used in Case 1
(1 kgf/cm² = 98 kPa, 1 kgf/cm²/cm = 9.8 MPa/m)

<table>
<thead>
<tr>
<th>Layer</th>
<th>( \gamma_{\text{m}} ) (kgf/cm²)</th>
<th>( \gamma_{\text{i}} ) (kgf/cm²)</th>
<th>( k_{\text{e}} ) (kgf/cm²/cm)</th>
<th>( K_{\text{e}} ) (kgf/cm²/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ac</td>
<td>0.1</td>
<td>0.0</td>
<td>0.5564</td>
<td>0.0</td>
</tr>
<tr>
<td>Tg</td>
<td>0.5</td>
<td>0.0</td>
<td>3.9935</td>
<td>0.0</td>
</tr>
<tr>
<td>S1</td>
<td>0.9</td>
<td>0.0</td>
<td>3.6293</td>
<td>0.0</td>
</tr>
<tr>
<td>C1</td>
<td>0.8</td>
<td>0.0</td>
<td>2.5465</td>
<td>0.0</td>
</tr>
<tr>
<td>C2</td>
<td>0.8</td>
<td>0.0</td>
<td>6.1566</td>
<td>0.0</td>
</tr>
<tr>
<td>S4</td>
<td>2.7</td>
<td>0.0</td>
<td>9.0766</td>
<td>0.0</td>
</tr>
<tr>
<td>S4 (Base)</td>
<td>( \sigma_{\text{h}} = 154 )</td>
<td>( \sigma_{\text{h}} = 154 )</td>
<td>( k_{\text{h}} = 53 )</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Soil parameters used in Case 2
(1 kgf/cm² = 98 kPa, 1 kgf/cm² = 9.8 MPa/m)

<table>
<thead>
<tr>
<th>Layer</th>
<th>( \gamma_{\text{m}} ) (kgf/cm²)</th>
<th>( \gamma_{\text{i}} ) (kgf/cm²)</th>
<th>( k_{\text{e}} ) (kgf/cm²/cm)</th>
<th>( K_{\text{e}} ) (kgf/cm²/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ac</td>
<td>0.05</td>
<td>0.05</td>
<td>0.5564</td>
<td>( K_{\text{e}} = k_{\text{h}} )</td>
</tr>
<tr>
<td>Tg</td>
<td>0.25</td>
<td>0.25</td>
<td>3.9935</td>
<td>( K_{\text{e}} = k_{\text{h}} )</td>
</tr>
<tr>
<td>S1</td>
<td>0.45</td>
<td>0.45</td>
<td>3.6293</td>
<td>( K_{\text{e}} = k_{\text{h}} )</td>
</tr>
<tr>
<td>C1</td>
<td>0.40</td>
<td>0.40</td>
<td>2.4385</td>
<td>( K_{\text{e}} = k_{\text{h}} )</td>
</tr>
<tr>
<td>C2</td>
<td>0.40</td>
<td>0.40</td>
<td>6.1566</td>
<td>( K_{\text{e}} = k_{\text{h}} )</td>
</tr>
<tr>
<td>S4</td>
<td>1.55</td>
<td>1.35</td>
<td>9.0766</td>
<td>( K_{\text{e}} = k_{\text{h}} )</td>
</tr>
<tr>
<td>S4 (Base)</td>
<td>( \sigma_{\text{h}} = 154 )</td>
<td>( \sigma_{\text{h}} = 154 )</td>
<td>( k_{\text{h}} = 53 )</td>
<td></td>
</tr>
</tbody>
</table>

The external and the internal lateral surfaces was identical. This assumption is acceptable, since effects due to curvature of the pile wall will be small, as suggested by Randolph (1987). The spring stiffness, \( k_{\text{e}} \), for the external shear stress was estimated from Eq. (40), and the spring stiffness, \( K_{\text{e}} \), for the internal shear stress was assumed to be equal to \( k_{\text{h}} \). The spring stiffness, \( K_{\text{h}} \), and the damping constant, \( C_{\text{h}} \), for the soil reaction of the base of the soil plug were estimated using Eq. (42) and Eq. (43), respectively, with the assumption that the soil reaction at the base of the soil plug was equivalent to the reaction of a half-space to the vertical motion of a rigid circular footing. The other soil parameters used in Case 2 were identical to those used in Case 1. The soil parameters used in Case 2 are summarized in Table 4.

**Analyses of Pile Driving**

The downward travelling stress waves at the pile head (Fig. 14) monitored from the method of two-point strain measurements were used as the boundary conditions at the pile head in the analyses of pile driving. This permitted us to bypass the modelling of the hammer action including gas forces, anvil and cushion. The evolution of the downward travelling stress at the pile head (Fig. 14) expressed a typical evolution of diesel hammer impact, including a gradual increase during the compression phase, a sharp increase at impact, gradual decrease during the expansion phase, and drop to zero at exhaust (Rempe and Davission 1977).

The stress waveforms at four different gauge points (No. 20, 17, 6 and 2) calculated in Case 1 are shown in Fig. 15. Those calculated in Case 2 are shown in Fig. 16. In the pile driving analyses, residual axial stresses in the pile and residual shear stresses along the pile shaft were not considered. In other words, shear stresses along the pile shaft before each driving were set to zero. Comparison of the recorded stress waveforms (Fig. 8) with the calculated waveforms in Fig. 15 and Fig. 16 shows that the waveforms calculated in Case 2 match the measured data better than those calculated in Case 1. It should be noted here that analyses of pile driving using the damping constants estimated from Eq. (41) were carried out in both Case 1 and Case 2; however, correlation of calculated and measured stress waveforms was less consistent for Case 2 waveforms shown in Fig. 16.

The peak amplitudes of the axial stresses along the pile calculated in Case 2 are compared with the measured data in Fig. 17. It was observed from the measured data that the peak amplitude of the stress tended to decay slightly as the base was reached and that sharp decay occurred in the lower four meters. The calculated results matched the measured...
results, including the sharp decay near the pile base. The calculation indicated that the sharp decay of the peak amplitude of the stresses at the lower four meters was due mainly to the superposition of downward travelling compression waves induced by the hammer striking at the pile head and the reflection of precursing tension waves at the pile base. On the other hand, such superposition of compression and tension waves did not occur above the lower four meters because of a relatively short duration of the incident stress at the pile head.

The time-displacement curve for the pile head calculated in Case 1 is shown in Fig. 18. The time-displacement curves for the pile head and the top surface of the soil plug calculated in Case 2 are shown in Fig. 19. The final set per blow of the pile calculated in Case 1 and Case 2 were not identical even though the total resistance in both cases were identical. Comparison of the monitored time-displacement curve of the pile head (Fig. 9) and the calculated results indicates that the time-displacement of the pile head calculated in Case 2 conformed well with the total behaviour of the monitored time-displacement curve, although the calculated result overestimated the final set per blow ($s_f=4$ mm). The calculated maximum displacement of the pile head of 11.5 mm and the final set of 4 mm (Case 2) gave the rebound per blow, $K$, equal to 7.5 mm, which agreed with the measured value ($K=7$ mm).
The calculated time-displacement curve of the top surface of the soil plug (Fig. 19) may be compared with the measured behaviour of the soil plug. In actuality, the top surface of the soil plug remained at the original ground surface even after a number of consecutive driving (1.1 m higher than the original ground surface, Fig. 7). The calculated final displacement of the soil plug was equal to zero, indicating that the top surface of the soil plug remained at the original ground surface after repeated driving even if internal shear stresses were operating along the interface between the soil plug and the inner lateral surface of the pile. It is interesting to note that the phase shift between the displacements of the pile and the soil plug can be seen in Fig. 19. The maximum displacement of the pile head occurred at $t=32$ ms, whereas the top surface of the soil plug attained the maximum displacement at $t=62$ ms.

The evolutions of the energy transmitted to the pile, the energy of the pile and the soil plug calculated in Case 1 and Case 2 are shown in Fig. 20 and Fig. 21, respectively. The energy...
of the pile, $W_P$, is the sum of the kinetic energy, $W_K$, and the strain energy, $W_S$:

$$W_K = A \int_0^L \{ \rho \dot{u}(x, t) \}^2 dx \quad (44)$$

$$W_S = \frac{A}{2E} \int_0^L [\sigma(x, t)]^2 dx \quad (45)$$

$$W_P = W_K + W_S \quad (46)$$

The energy of the soil plug, $W_{PL}$, which is the sum of the kinetic energy, $W_{PLK}$, and the strain energy, $W_{PLS}$, is similarly calculated as follows:

$$W_{PLK} = \sum_{i=0}^{n} M_i V_i^2 \quad (47)$$

$$W_{PLS} = \frac{1}{2} \sum_{i=1}^{n} B_i C_i^2 \quad (48)$$

$$W_{PL} = W_{PLK} + W_{PLS} \quad (49)$$

where $M_i$ is the mass, $V_i$ is the velocity of the $i$-th node of the soil plug, $B_i$ is the spring constant and $C_i$ is the compression of the $i$-th element of the soil plug.

The energy, $W$, transmitted to the pile attained the peak value at time $t=34$ ms in both cases. The final energy transmitted to the pile calculated in Case 2 was 687 tf cm (67.3 kN m), which was in good agreement with the monitored value of 685 tf cm (68.1 kN m, Fig. 10), whereas the final value calculated in Case 1 was 517 tf cm (50.7 kN m).

The energy of the pile, $W_P$, rapidly decreased after time $t=25$ ms and fell to only 5% of the final value of the energy transmission in both cases. The energy of the soil plug, $W_{PL}$, attained only a small amount of the energy transmitted to the pile.

The elastic strain energy of the soil, which corresponds to the strain energy of the spring component in the soil model, and the plastic energy consumed by the plastic slider also were calculated. The calculated results showed that the value of the elastic strain energy of the soil was only a negligible amount of the energy transmitted to the pile and that 91% of the energy transmitted to the pile was consumed by the plastic slider component. Note that the radiation energy was equal to zero, since the radiation damping was taken as zero in the calculations.

The final set per blow of the pile head calculated in Case 1 and Case 2 were not identical (Fig. 18 and Fig. 19) even though the total resistance for both cases were identical. From comparison of Fig. 18 through Fig. 21, this may be attributed to the final value of the energy transmitted to the pile from the hammer.

The changes over time of the distributions of the external and the internal shear stresses (Case 2) are shown in Fig. 22. At time $t=$
30 ms, the distributions of the internal and the external shear stresses along the pile were almost identical. The time $t=30$ ms corresponded to the time when the peak incident stress reached the pile base and the pile head attains the maximum displacement (Fig. 19). Note that the shear stresses along the layers, $A_1$, $T_1$, $S_1$ and $C_1$ attained the maximum values, whereas the shear stress along the layers, $C_2$ and $S_4$, did not reach the maximum value at $t=30$ ms.

At a time $t=40$ ms, the internal shear stresses along the pile except for layer $S_4$ were negative (acting downwards to the pile and acting upwards to the soil plug), whereas external shear stresses were positive along the all layers. At this time, the external maximum shear stresses along layers $C_2$ and $S_4$ were developed. The total shaft force including the internal and the external shear stresses is only 138 tf (1.35 MN) because the internal and the external shear stresses were acting in opposing directions (total internal shaft force = $-225$ tf ($-2.21$ MN), total external shaft force $=363$ tf (3.56 MN)). The phase shift between mobilization of the internal shear stress and the external shear stress well corresponds to the phase shift between the displacements of the pile and the soil plug shown in Fig. 19. At time $t=50$ ms, both internal and external shear stresses along layers $C_1$ and $C_2$ fell to zero.

**Analyses of Static Loading Test**

The static loading test described in the previous section (Fig. 11 and Fig. 12) were analyzed.

The results of the static loading test calculated in Case 1 and Case 2 are compared with the measured data in Fig. 23 and in Fig. 24, respectively. There was relatively good agreement in Case 2 between the calculated results and the measured data for both the pile head and the pile base compared to Case 1, although the bearing capacities calculated in both cases were identical.

The distribution of the axial forces calculated in Case 2 are compared with the measured data in Fig. 25. It can be seen that the calculated results are in good agreement with the measured data, especially at the final loading stage.

The distribution of internal and external shear stresses along the pile calculated in Case 2 are shown in Fig. 26. With the increase in pile head force, external shear stresses developed more rapidly than internal shear stresses, although the distribution of external and internal shear stresses were almost iden-
CONCLUSIONS

Stress wave theory was applied to the offshore open-ended pipe pile to clarify the effects of the soil plug on the behaviour of the pile during driving. The interaction between the pile and the soil plug as well as the external soil was incorporated in the analyses of the pile driving and the static loading test. Calculated results conformed well with the measured piling performance during driving and static loading.

The main conclusions drawn from the present analyses are as follows:

1. Final set per blow of a pile for a given hammer is changed by the distribution of soil resistance even if the total resistance is identical. This is correlated to the energy finally transmitted to the pile from the hammer.

2. Mobilization process of internal shear stresses and external shear stresses during driving are not identical. This may be attributed to the phase shift of the displacements between the pile and the soil plug.

3. Only imperfect plugging of the pipe pile occurs during driving.

4. In the analysis of the static loading test, with the increase in the pile head
force, external shear stresses develop more rapidly than internal shear stresses, although the distribution of external and internal shear stresses are almost identical at the final loading stage. Side failure at the internal and the external pile-soil interfaces progresses from the head and the base of the pile toward its centre.

Comparison with full scale measurements are necessary to examine the validity of the above conclusions. Measurements of acceleration of the top surface of soil plug is relatively feasible and will give us valuable information on the behaviour of soil plug.

In the present analyses of pile driving, the coefficient of the radiation damping was taken as zero and other soil parameters were taken from the results of the static loading test. Determination of appropriate soil parameters used in the analyses of pile driving and static loading test needs future studies.

ACKNOWLEDGEMENTS

The authors are indebted to Prof. Hideki Ohta, Kanazawa University, and Associate Prof. Hideo Sekiguchi, Kyoto University, for helpful discussions during this study. To Kansai International Airport Co. Ltd., we extend our grateful appreciation for his permission to use the field data for this study.

NOTATIONS

\(A=\) cross sectional area of pile
\(A_s=\) cross sectional area of soil plug
\(b_m=\) spring constant of \(m\)-th element of pile
\(B_m=\) spring constant of \(m\)-th element of soil plug
\(c=\) bar wave velocity of pile
\(c_b=\) damping constant for pile base resistance
\(c_s=\) damping constant for base of soil plug
\(C_m(t)=\) compression of \(m\)-th element of soil plug
\(c_s=\) damping constant for external shear stress
\(C_s=\) spring constant for internal shear stress
\(d_s=\) spring stiffness for base force of pile
\(D_b=\) spring stiffness for base force of soil plug
\(d_m=\) spring stiffness for external shear force at \(m\)-th node
\(D_m=\) spring stiffness for internal shear force at \(m\)-th node

\(E=\) Young’s modulus of pile
\(E_{os}=\) constrained Young’s modulus of soil
\(f_m(t)=\) downward travelling stress-wave at \(m\)-th node at time \(t\)
\(F_m(t)=\) total force acting on \(m\)-th node at time \(t\)
\(g_m(t)=\) upward travelling stress-wave at \(m\)-th node at time \(t\)
\(G_s=\) shear modulus of soil
\(j_m=\) damping constant for external shear force at \(m\)-th node
\(J_m=\) damping constant for internal shear force at \(m\)-th node
\(k_s=\) spring stiffness for base resistance of pile
\(K_s=\) spring stiffness for base resistance of soil plug
\(k_s=\) spring stiffness for external shear stress
\(K_s=\) spring stiffness for internal shear stress
\(L=\) length of pile
\(N_m(t)=\) axial force of \(m\)-th element of soil plug at time \(t\)
\(P_o=\) pile head force in static loading test
\(r_i=\) inner radius of pile
\(r_m=\) external shear force acting on \(m\)-th node of pile
\(R_m=\) internal shear force acting on \(m\)-th node of pile
\(r_o=\) outer radius of pile
\(t=\) time
\(u=\) axial displacement of pile
\(u_s=\) axial displacement of external soil adjacent to pile
\(U=\) axial displacement of soil plug adjacent to pile
\(U=\) axial displacement of soil plug at centre
\(v=\) particle velocity of pile
\(V_m(t)=\) particle velocity of \(m\)-th node of soil plug
\(W=\) energy transmitted to pile
\(W_k=\) kinematic energy of pile
\(W_p=\) energy of pile \((= W_k+W_s)\)
\(W_s=\) strain energy of pile
\(x=\) axial co-ordinate along pile
\(Z_m(t)=\) accelerating force at \(m\)-th node of soil plug
\(\Delta t=\) timestep
\(\rho=\) mass density of pile
\(\rho_s=\) mass density of soil
\(\alpha=\) axial stress in pile
\(\alpha_b=\) base resistance of pile
\(\alpha_s=\) base resistance of soil plug
\(\tau=\) shear stress at pile-soil interface
\(\tau_{max}=\) maximum shear stress at pile-soil interface

(Suffix)

\(b=\) subscript to identify quantities associated with bases of pile and soil plug
\(m=\) subscript to identify quantities associated with \(m\)-th nodes or \(m\)-th elements of pile and soil plug
REFERENCES


23) Novak, M., Nogami, T. and Aboul-Ella, F.
EFFECTS OF SOIL PLUG


\[ b_m = AE/\Delta L \] (A 2)

The equilibrium of the force at m-th node is expressed as

\[ \Delta n_m = \Delta n_{m-1} - (d_{m-1} \cdot \Delta u_{m-1} + D_{m-1} \cdot (\Delta u_{m-1} - -U_{m-1})) \] (A 3)

where \( U_m \) is the displacement of the m-th node of the soil plug at the centre.

Combination of Eq. (A 1) and Eq. (A 3) leads to

\[ b_m \cdot \Delta u_{m-2} - (b_{m-1} + d_{m-1} + D_{m-1} + b_m) \Delta u_{m-1} + D_{m-1} \cdot \Delta u_{m-1} + b_m \cdot \Delta u_m = 0 \] (A 4)

The equilibrium of the force at the pile base \((m=n)\) is given by

\[ \Delta n_n = b_n (\Delta u_{n-1} - \Delta u_n) \] (A 5)

Then, it follows that

\[ b_n \cdot \Delta u_{n-1} - (b_n + d_n) \Delta u_n = 0 \] (A 6)

The equilibrium of the force at the pile head \((m=0)\) is given by

\[ \Delta P_0 = \Delta n_1 + d_1 \cdot \Delta u_0 + D_0 \cdot (\Delta u_0 - U_0) = b_1 (\Delta u_0 - U_1) + d_1 \cdot \Delta u_0 + D_0 (\Delta u_0 - U_0) \] (A 7)

where \( \Delta P_0 \) is the increment of the load applied to the pile head. Then,

\[ - (d_0 + D_0 + b_1) \Delta u_0 + D_0 \cdot \Delta u_0 + b_1 \cdot \Delta u_1 = - \Delta P_0 \] (A 8)

The increment of axial force, \( \Delta N_m \), of m-th element of the soil plug is given by

\[ \Delta N_m = B_m (\Delta U_{m-1} - \Delta U_m) \] (A 9)

The equilibrium of the force at m-th node of the soil plug is given by

\[ \Delta N_m = \Delta N_{m-1} - D_{m-1} (\Delta u_{m-1} - \Delta u_{m-1}) \] (A 10)

Combination of Eq. (A 9) and Eq. (A 10) leads to

\[ B_{m-1} \cdot \Delta U_{m-2} + D_{m-1} \cdot \Delta U_{m-1} + (B_{m-1} + D_{m-1} + B_m) \Delta U_{m-1} + B_m \cdot \Delta U_m = 0 \] (A 11)

The equilibrium of the force at the bottom of the soil plug \((m=n)\) is given by

\[ \Delta N_n = B_n (\Delta U_{n-1} - \Delta U_n) \] (A 12)

Then,

\[ B_n \cdot \Delta U_{n-1} - (B_n + D_n) \Delta U_n = 0 \] (A 13)

In the above equations, \( d_m \), \( D_m \), \( b_n \) and \( D_n \) should be referred to Eqs. (3), (7), (11) and

APPENDIX

A matrix method is used to calculate the displacement, axial force and shear stresses at the pile-soil interfaces at each point of the pile.

The increment of axial force, \( \Delta n_m \), of m-th element of the pile is given by

\[ \Delta n_m = b_m (\Delta u_{m-1} - \Delta u_m) \] (A 1)

where \( b_m \) is the spring constant defined by

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(15), respectively. They are taken as zero when the maximum shear stress, $\tau_{\text{max}}$, or the maximum point-bearing resistance, $\sigma_{\text{max}}$, are reached at each node.

The equations (A.4), (A.6), (A.8), (A.11) and (A.13) are assembled in the form:

$$[K] \{\Delta u\} = \{\Delta F\} \quad (A.14)$$

where $K$ is the $2(n+1) \times 2(n+1)$ symmetric matrix assembling the coefficients, and $\{\Delta u\}$ and $\{\Delta F\}$ are the column vectors defined as follows:

$$\{\Delta u\}^T = \{\Delta u_0, \Delta U_0, \Delta u_1, \Delta U_1, \ldots, \Delta u_n, \Delta U_n\} \quad (A.15)$$

$$\{\Delta F\}^T = \{-\Delta P_0, 0, \ldots, 0\} \quad (A.16)$$

If $\Delta P_0$ is given, $\{\Delta u\}$ are solved from Eq. (A.14). Increments of quantities such as axial forces of the pile and soil plug and reaction forces of soils acting on pile nodes are obtained by substituting the known increments of the displacements into the corresponding equations.